Chinese Science Bulletin 2003 Vol. 48 No.10 1034—1039

Global analysis theory of climate system and its applications

LI Jianping[1,2] & CHOU Jifan[2,3]

1. LASG, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China;
2. Department of Atmospheric Sciences, Lanzhou University, Lanzhou 730000, China;
3. Training Center for China Meteorological Administration, Beijing 100081, China

Correspondence should be addressed to Li Jianping (e-mail: ljp@lasg.iap.ac.cn)

Abstract **The idea and main theoretical results of the global analysis theory of climate system are briefly summarized in this paper. A theorem on the global behavior of climate system is given, i.e. there exists a global attractor in the dynamical equations of climate, any state of climate system will be evolved into the global attractor as time increases, indicating the nonlinear adjustment process of climate system to external forcing. The different effects of external forcing, dissipation and nonlinearity on the long-term behavior of solutions are pointed out, and some main applications of the global analysis theory are also introduced. Especially, three applications, the adjustment and evolution processes of climate, the principle of numerical model design and the optimally numerical integration, are discussed.**

Keywords: climate system, global analysis, global attractor, principle of operator constraint, computational uncertainty principle.

DOI: 10.1360/02wd0369

Climate science is at present one of the most concerned sciences in the world and also a hot issue[1,2]. This may be clearly seen from the famous world climate research, the Climate Variability of Predictability (CLIVAR). Climate is a physical system and satisfies some general conversation laws in physics, e.g. the mass conversation, energy conversation, momentum conservation, water mass conservation, etc. In the light of these principles, the evolutionary equations of climate system can be deduced as a set of partial differential equations by use of proper mathematical analysis, and according to circumstance and historic conditions of the system, its initial value and boundary value conditions are given correctly, then a set of full dynamical equations of climate will be determined. The studies of climate dynamics, numerical simulation and dynamical diagnostics just involve the equations and their various simplified forms[3—8]. However, they are very complex nonlinear equations with dissipation and external forcing. Nonlinearity and complicated processes related to phase change of latent heating in climate are two basic difficulties for studying dynamical behavior of climate[9—13]. The traditional climate dynamics is limited in the frames of linear system or nonlinear conservative system, due to the lack of effective mathematical theories and methods for dealing with this kind of system. This is essential drawback for the climate system whose evolutionary time scale is far larger than the time scale of energy dissipation. After all, climate is an open system and also a dissipative structure that exchanges energy continuously with outside. Modern climate dynamics therefore calls for new dynamical theory——the forced dissipative nonlinear dynamics of climate[9—31].

The time scale of evolution of climate system is larger than that of dissipation, it is, therefore, important that the dynamic property and basic characteristics of long-term evolution of climate system are explored by dynamic theory. This is one of the most important and fundamental problems in climate dynamics with both significant theoretical sense and important practical value. This issue simply needs to understand global asymptotic behavior of solutions in mathematics, which needs to be studied by the global qualitative theory of differential equations. The global analysis studies global behavior and whole characteristics of all possible initial values of system after time goes arbitrary long time, especially, discusses final state of the system while time trends infinite. Although a more strictly description of final state in mathematics is for the case of infinite time, in fact this restriction is not necessary in practice. For the practical problem it is significant to study the state arrived after definitely finite time. The global characteristics of solutions should be studied by use of the global analysis theory since for the traditional methods, e.g. analytical method, numerical method and experiment method, there are some difficulties that cannot be overcome in analyzing this subject. The global analysis is a direct method, that is, asymptotic properties of solutions are studied directly through characteristics and properties of equations themselves and this does not need to solve solution. It uses some theories and methods of infinite dimensional dynamical system, its purpose is to understand universal characteristics of final state of the system, explain climate phenomena, illustrate climate regulation, design new numerical scheme and computational method, etc. This report mainly tries to summarize some important achievements and new findings in the global analysis theory of climate from Chinese scientists and to introduce some applications of these theoretical results.

1 Main idea and theoretical results of the global analysis

(ⅰ) Main idea. As known, climate is a complicated physical system with both various forms and omnifarious local properties. The global analysis simply leaves aside various concentrate motions and local properties of climate and investigates whole characteristics and global behavior of climate. This method can refine essential relations between different physical variables in climate system and clarify basic influences of external forcing on climate motion, then reveals the most fundamental regulations of climate system. These general principles from the global analysis may be used to study particularity of particular motion and to guide specific practice. Thus, the global analysis is very important not only for theoretical research on climate dynamics, but also for numerical climatic simulation and prediction. Besides, the global analysis is also to study global characteristics of climatic dynamical equations as complete as possible, one of reasons is to provide a constraint principle through theoretical results to design variously proper simplifications and discretization of equations which does not distort the most basically physical laws of original system.

The general steps of the global analysis are as follows: the original dynamical equations should be deduced first to be an equivalent operator equation in a certain Hilbert space, and then through analyzing properties of operators in the equation and their respective physical sense the “goal” (final state) of all points and related questions are studied in the Hilbert space.

(ii) Operator equation. All of the full atmospheric equations (whether it is the dry air or the moist atmosphere, and the equations contain all kinds of motion), the full ocean equations (the state equation in them is very general, i.e. $f(\rho, p, S, T) = 0$), the coupled air-sea equations and the ocean-land-atmosphere equations can be written as the following equivalent operator equation in the Hilbert space[9—29]1):

$$\begin{cases} \dfrac{\partial \varphi}{\partial t} + [N(\varphi) + L(\varphi)]\varphi = \xi(\varphi), \\ \varphi\big|_{t=0} = \varphi_0, \end{cases}$$

where the abstract operators $N(\varphi)$ and $L(\varphi)$ have basic properties as follows:

$$N(\varphi) = -N^*(\varphi),\ (N(\varphi)\varphi_1, \varphi_2) = -(\varphi_1, N(\varphi)\varphi_2),$$
$$(N(\varphi_1)\varphi, \varphi) = 0;\ L(\varphi) = L^*(\varphi),$$
$$(L(\varphi)\varphi_1, \varphi_2) = (\varphi_1, L(\varphi)\varphi_2), \quad (L(\varphi)\varphi, \varphi) \geqslant 0,$$

$\forall \varphi, \varphi_1, \varphi_2 \in H_0(\Omega)$. $N^*(\varphi)$ and $L^*(\varphi)$ are the adjoint operators of $N(\varphi)$ and $L(\varphi)$ respectively, and $H_0(\Omega)$ is a complete Hilbert space with the following inner product and norm:

$$(\varphi_1, \varphi_2) = \int_\Omega \varphi_1' \varphi_2 \mathrm{d}\Omega = \int_0^{2\pi} \int_0^{\pi} \int_{r_s}^{r_\infty} \varphi_1' \varphi_2 r^2 \sin\theta \mathrm{d}r \mathrm{d}\theta \mathrm{d}\lambda.$$
$$\|\varphi\|_0 = (\varphi, \varphi)^{1/2},$$

where λ, θ, r are the longitude, co-latitude and geocentric distance, respectively. The basic physical senses implied by the above properties of the operators $N(\varphi)$ and $L(\varphi)$ are that the former represents all kinds of reversible adiabatic processes of energy conversation and the latter represents the irreversible diabatic processes of energy dissipation. The properties of the operators $N(\varphi)$ and $L(\varphi)$ reflect the essential characteristics of two kinds of basically physical process with entirely distinct physical means.

(iii) Global behavior theorem. The global analysis theory[9—29] proves that there exists a global attractor in the dynamical equations of climate (whether it is under the condition of stationary or non-stationary external forcing), and any state of climate system will evolve into the global attractor as time increases. The state out of the attractor only possesses transient sense. The global attractor possesses finite dimension and represents the final state of system. This reveals a very important process of climate, the nonlinear adjustment to the external forcing.

The global behavior theorem shows that the long-term behavior of climate system can be described by a finite dimensional dynamical system. This just implies a physical and mathematical base of the theory that climate as an infinite dimensional system can be simulated and predicted numerically by a finite dimensional dynamical system[9—29].

(iv) Interactions among external forcing, dissipation and nonlinearity. The global analysis theory suggests that the joint action of nonlinearity, dissipation and external forcing is the source of atmospheric multiple equilibria, i.e. the phenomenon of atmospheric multiple equilibria is a kind of nonlinear mechanism with interactions between dissipation and external forcings[23].

Effects of external forcing, dissipation and nonlinearity on the long-time behavior of solutions are essentially different. The existence of global attractor indicates a fundamental distinction between chaos of dissipative system and that of conservative system. The external forcing is a necessary condition for keeping activation of dissipative system. The dissipation is a necessary condition for holding overall stability for system with external forcing. The nonlinear is a necessary condition for all chaos systems. Therefore, a simplified dynamical model used to describe climate phenomenon must be a set of forced, dissipative and nonlinear evolutionary equations, neither adiabatic nor linear[12,23].

2 Main applications of the global analysis theory

There are many applications of the global analysis theory, for instance, to clarification of the adjustment and evolution processes of climate system, simplification of climatic dynamical equations, designing principle of differential scheme, analysis of nonlinear numerical stability, optimal numerical integration, principle of splitting algorithm, explanation of the essence of four-dimensional as-

1) Li Jianping, Qualitative theory of the dynamical equations of atmospheric and oceanic motion and its applications, Lanzhou University, Ph.D Dissertation, 1997, 209.

similation, designing rule of long-term forecasting model, use of preferred modes, etc.[9—29]. Owing to the limitation of space, three important applications in them will be introduced only as follows.

3 Adjustment and evolution processes of climate system

An important conclusion of the global analysis theory is that it reveals a nonlinear adjustment process of climate as a forced, dissipative and nonlinear system to external forcing. This kind of adjustment process is essentially different from the previous adjustments in the traditional dynamical meteorology such as the geostrophic adjustment, rotational adjustment, potential vorticity adjustment, static adjustment, etc. The existence of global attractor is simply the difference between the adjustment of climate system and the above adjustment.

It follows from the global behavior theorem that there exists a global attractor in the climate system, and any state out of the global attractor must be attracted to the attractor. The global attractor is contained in a global absorbing set which is a point set with finite radius. Any state out of the absorbing set will be attracted into the absorbing set after finite time, and any point in the set moves in it forever and cannot run out of the set[25]. If one uses the method of smooth inertial manifold to analyze the approximation process from out of the inertial manifold to the global attractor smooth inertial manifold (the global attractor is on the inertial manifold), the approximation speed is exponentially[27]. Hence, this kind of adjustment to the final state determined by given external forcing is a very fast process. The state out of the global attractor possesses only transient sense. The attractor is an invariant set and has the property of relative stability and may be regarded as a kind of equilibrium, so the evolution of state on the attractor is a slow process. It may be concluded that there exist two kinds of characteristic time scales, the fast adjustment process to the attractor and the slowly evolutionary process on the attractor, while the external forcing is stationary or its variation is very slow. Moreover, there exists the third time scale, the more slowly evolutionary process of macroscopic state versus external parameters, when the external forcing is varied.

Zeng[4,33,34] presented first the concept of time boundary layer (TBL) when he discussed the adjustment and evolution processes in the adiabatic non-frictional atmosphere. Here we use the global analysis theory to extend this concept to the forced, dissipative, nonlinear climate system. According to the properties and characteristics of climate motion, three classes of TBLs, the first, second and third TBLs, can be introduced. The adjustment and evolution processes of climate system can be understood clearly by use of the three kinds of TBLs. As shown in Fig. 1, the first TBL is a quick adjustment process of the state out of the attractor to the attractor, and is corresponding to the first time scale mentioned before. The outside of the first TBL is the evolution process on the attractor, and is corresponding to the second time scale mentioned above, and the corresponding system is a forced, dissipative, nonlinear system with stationary external forcing. The second TBL is a layer including the first TBL and the evolution process outside of the first

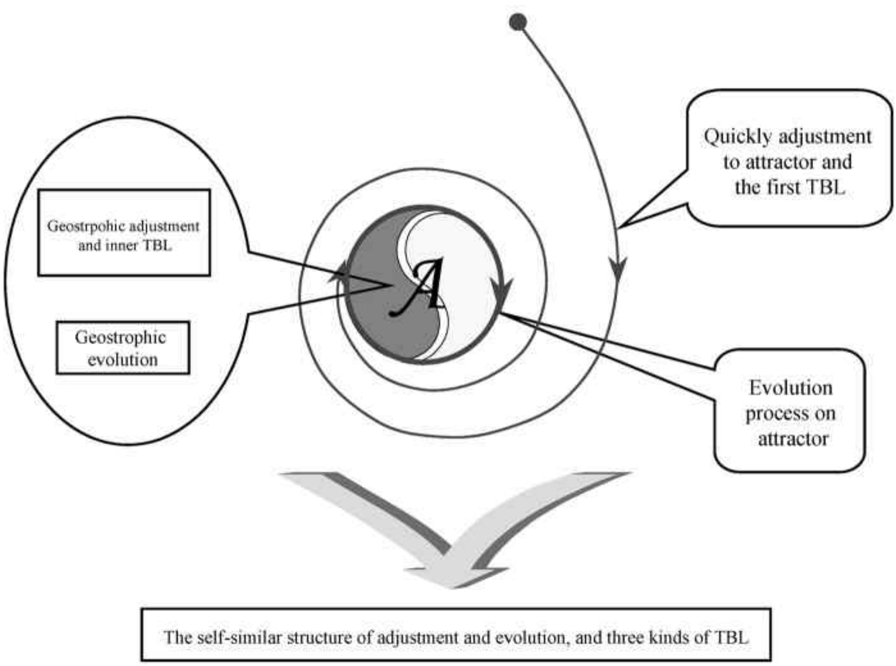

Fig. 1. The adjustment and evolution processes of climate system.

TBL. The outside of the second TBL is a more slow evolution process of macroscopic state against external forcing, corresponding to the third time scale stated previously, meanwhile the corresponding system should be regarded as a forced, dissipative, nonlinear system with non-stationary external forcing. Besides, within the time scale of dissipation the atmosphere may be regarded as an adiabatic non-frictional system, and there are the geostrophic adjustment and evolution processes as usual. The TBL in this case is the inner TBL (i.e. the third TBL), and the corresponding system is the adiabatic non-frictional conservative system. The inner TBL here is equivalent to the TBL concept from Zeng[4,33,34]. The above characteristics reflect a kind of self-similar structure of TBL for the forced, dissipative, nonlinear system. Based on the concept of the three classes of TBL, the adjustment and evolution processes of forced, dissipative, nonlinear climate system may be understood clearly, and also be applied to design of splitting algorithm of numerical model[22]. However, the lengths of the above TBLs and the corresponding characteristic parameters are still needed to further analysis from physics.

4 Principle of operator constraint for designing numerical model[9,26]

The principle of design of numerical model plays a key role in making the numerical prediction model. This can be seen from the following facts. Richardson[35] failed to do the primitive equation model since he did not know how to simplify equations. Charney[36] presented the scale analysis and applied it successfully to the barotropic vorticity model to make the first 500 hPa numerical prognostic chart[37]. Then Lorenz[38] presented the energy constraint principle and Shuman and Hovermale[40] applied this rule successfully to the primitive equation model. However, except the scale analysis and the energy constraint, any new rule has not suggested to design numerical model since Shuman and Hovermale[39] set up the primitive equation model. But the two principles only adapt to the case of adiabatic non-frictional conservative system with static approximation and do not adapt to the cases of diabatic and dissipation. As indicated by Liao[40], this is unfitting to the main task for designing the primitive equation model that is how to design better the nonlinear system with external forcing and dissipation. In the light of the global analysis theory a new principle, the operator constraint method[9,26], can be presented. The core of this principle is just that it can keep the consistency of basic properties of the corresponding operators in the equations before and after their simplification. This method is an extension of the energy constraint rule and has both strict mathematical base and clear physical sense. It can guarantee that the simplified equations from it do not distort the global properties and physical essence of the original equations. This method aims at the case of diabatic, non-static and dissipation, it therefore is suitable for the model design of forced, dissipative, nonlinear system.

5 Optimal numerical integration[31,32]

Dynamical equations of climate must be discretized while climate system is numerically simulated and forecasted. We always hope to have optimal numerical integration in the process of obtaining numerical solutions. The optimal here aims at computational accuracy. The optimal numerical integration is simply the method that it can make a certain numerical method to carry out calculation in its best degree of accuracy. For the given equations and numerical method, spatial and time resolution and machine precision are the primary parameters which can determine the best accuracy of the method, if the physical model considered has not a model error. An optimal method of numerical integration can be found by use of the global analysis.

According to some numerical experiments[31] we found the following qualitative conclusion for numerical error and effective computation time (ECT) profile (Fig. 2): The total error will initially decrease as stepsize decreases and the discretization error decreases, so that the ECT increases; as stepsize decreases to a certain extent, however, the round-off error becomes dominant because the iterative times increase, as a result, the total error increases and the ECT decreases. The inevitable consequence is that there exists a stepsize H (Fig. 2), and then at this stepsize the total error is smallest and the ECT is largest. This stepsize H is therefore called the optimal stepsize. Owing to the inverse variation relation against stepsize between the discretization error and the round-off

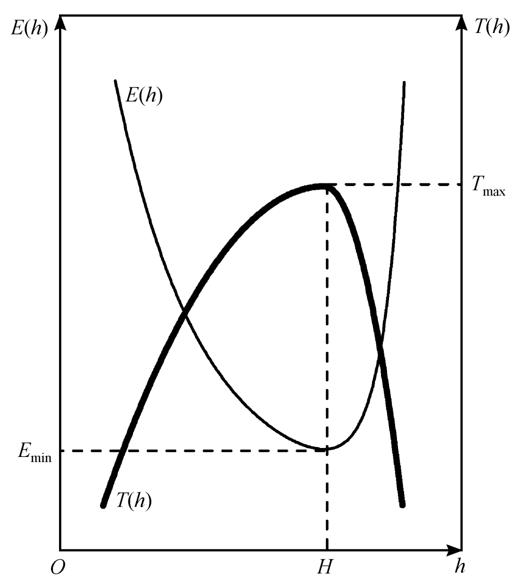

Fig. 2. Illustration of total error $E(h)$ and effective computation time $T(h)$ versus stepsize h. H denotes the optimal stepsize, $E_{\min}$ the minimum error and $T_{\max}$ the maximum ECT.

error, a computational uncertainty principle similar to the well-known Heisenberg uncertainty relation of quantum mechanics[12] is led. Specifically, if the discretization and round-off errors are treated as two “adjoint variables”, the computational uncertainty principle indicates that the smaller one of them, the greater the other adjoint variable will be. This means that once the precision of calculation machine used is given, the best degree of accuracy which can be achieved for the numerical solution obtained by a numerical method is determined entirely. The computational uncertainty principle gives a certain limitation to the maximum ECT of long-time numerical integration for chaos system which is sensitive to initial values and for some systems with transient chaos. This principle implies that the computational capacity of any numerical method is very limited for nonlinear system under the inherent property of finite machine precision.

To achieve optimal numerical integration, at first we must study systemically error formula through the global analysis, determine optimal relation from the analysis and find optimal spatial and time resolution and estimate maximum ECT. Then some necessary parameters in numerical integration can be given.

For ordinary differential equations, our numerical results and theoretical analysis show that there are two universal relations which are dependent only on machine precision and the order of numerical method and are independent of types of differential equations, initial values and numerical schemes[32], i.e.

$$l = \frac{H_1}{H_2} = 10^{\frac{n_2 - n_1}{p+0.5}},$$

$$k = \frac{C(T_2)}{C(T_1)} = l^p,$$

where H_i is the optimal stepsizes, n_i the significant digits of machines, p the order of the numerical method, $\boldsymbol{T}_i$ the maximum ECTs, i = 1, 2 represents two machines with different precision. Refs. [31] and [32] verified that the theoretical results from the two universal relations were in good agreement with those from the numerical experiments.

Based on the computational uncertainty principle and results mentioned above, we present an optimal integration method, the step-by-step adjustment method. The flow diagram of the method is shown in Fig. 3. Our numerical experiments verify that the step-by-step adjustment method is a very effective method for achieving optimal numerical integration. It may be expected that the method will be able to exhibit its extensive applications in practice.

6 Summary

The idea, purpose and main theoretical results of the global analysis theory of climate system are briefly summarized in this paper. Some applications of the theory are introduced, especially, three important applications, the adjustment and evolution processes of climate, the principle of numerical model design and the optimally numerical integration, are discussed. The new findings and achievements in the field from Chinese scientists are emphasized. It should be pointed out that the global analysis theory of climate is still under way, and many works in the area await further study. According to the current results

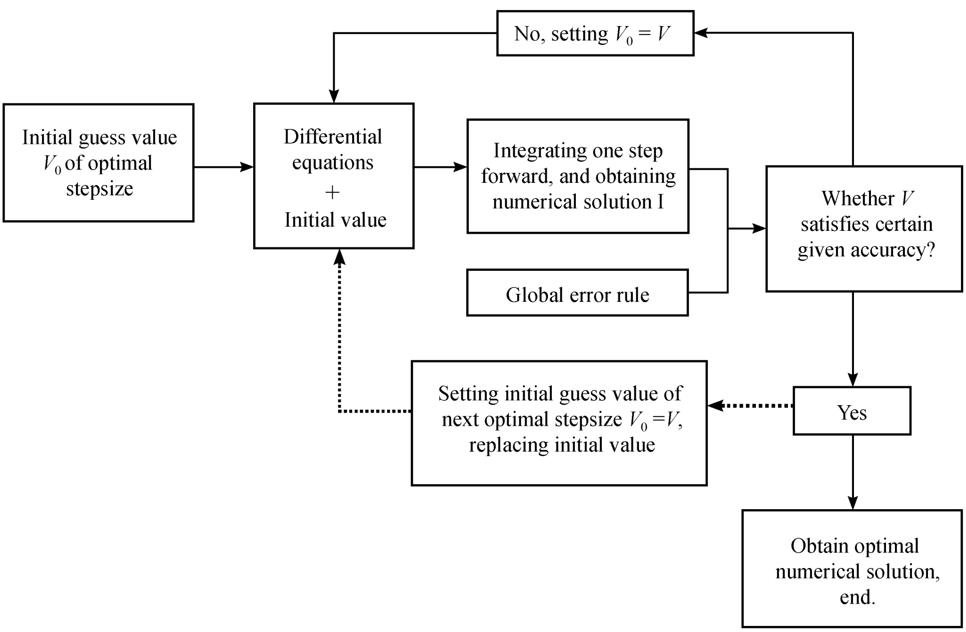

Fig. 3. The flow diagram of the step-by-step adjustment method of optimal numerical integration.

and their potential practical value, we believe that with further investigation the global analysis theory will play more important roles in the research of climatology in the future.

Acknowledgements This work was supported jointly by the National Key Basic Research Project (Grant No. G1998040901), the CAS Project (Grant No. ZKCX2-SW-210) and the National Natural Science Foundation of China (Grant Nos. 40135020 and 40275025).

References

1. Ye Duzheng, Zeng Qingcun, Guo Yufu, Current Research in Climate (in Chinese), Beijing: China Meteorological Press, 1991, 353.
2. World Climate Research Programme (WCRP), CLIVAR Initial Implementation Plan, WCRP-103, WMO/TD 869, ICPO 14, Geneva, Switzerland, 1998, 314.
3. Zeng Qingcun, Mathematical and Physical Foundations of Numerical Weather Prediction (in Chinese), Beijing: Science Press, 1979, 548.
4. Zeng Qingcun, A mathematic model of climate dynamics suitable for modern mathematical analysis, Chinese J. Atmos. Sci. (in Chinese), 1998, 22(4): 408—417.
5. Chou Jifan, Long-Term Numerical Weather Prediction (in Chinese), Beijing: China Meteorological Press, 1986, 329.
6. Yang Dasheng, Liu Yubin, Liu Shishi, Dynamical Meteorology (in Chinese), Beijing: China Meteorological Press, 1983, 82—99.
7. Liao Dongxian, Wang Liangming, Principles of Numerical Prediction and their Applications (in Chinese), Beijing: China Meteorological Press, 1986, 1—28.
8. Liu Shida, Liu Shishi, Atmospheric Dynamics (First Volume) (in Chinese), Beijing: Peking University Press, 1991, 1—33, 167—184.
9. Chou Jifan, Some properties of operators and the decay of effect of initial condition, Acta Meteor. Sinica (in Chinese), 1983, 41(4): 385—392.
10. Chou Jifan, New Advances in Atmospheric Dynamics (in Chinese), Lanzhou: Lanzhou University Press, 1990, 214.
11. Chou Jifan, Some advances and trends in the atmospheric dynamics, In The Forward Position and Prospects for Modern Atmospheric Sciences (in Chinese), Beijing: China Meteorological Press, 1995, 71—75.
12. Li Jianping, Chou Jifan, The effects of external forcing, dissipation and nonlinearity on the solutions of atmospheric equations, Acta Meteor. Sinica, 1997, 11(1): 57—65.
13. Li Jianping, Chou Jifan, Further study on the properties of operators of atmospheric equations and the existence of attractor, Acta Meteor. Sinica, 1997, 11(2): 216—223.
14. Wang Shouhong, Huang Jianping, Chou Jifan, Some properties of solutions for the equations of large scale atmosphere: Nonlinear adjustment to the time independent external forcing, Science in China, Ser. B, 1990, 33: 476—485.
15. Lions, J. L., Temam, R., Wang, S., Geostrophic asymptotics of the primitive equations of the atmosphere, Topological Methods in Nonlinear Analysis, Special Issue Dedicated to Jean Leray, 1994, 4: 253—287.
16. Lions, J. L., Manley, O., Temam, R. et al., Physical interpretations of the attractor for a simple model of atmospheric circulation, J. Atmos. Sci., 1997, 54(9): 1137—1143.
17. Temam, R., Wang, S., Mathematical problems in meteorology and oceanography, Bull. Amer. Meteorol. Soc., 2000, 81(2): 319—321.
18. Lions, J. L., Temam, R., Wang, S., New formulations of the primitive equations of atmosphere and applications, Nonlinearity, 1992, 5: 237—228.
19. Wang, S., Attractors for the 3D baroclinic quasi-geostrophic equations of large scale atmosphere, J. Math. Anal. Appl., 1992,165: 266—283.
20. Lions, J. L., Temam, R., Wang, S., Mathematical models and mathematical analysis of the Ocean/Atmosphere system, C. R. Acad. Sci. Paris, Ser I, 1993, 316: 211—215.
21. Lions, J. L., Temam, R., Wang, S., On the equations of large-scale ocean, Nonlinearity, 1992, 5: 1007—1053.
22. Li Jianping, Chou Jifan, The qualitative theory on the dynamical equations of atmospheric motion and its applications, Chinese J. Atmos. Sci. (in Chinese), 1998, 22(4): 348—360.
23. Li Jianping, Chou Jifan, Source of atmospheric multiple equilibria, Chinese Science Bulletin, 1996, 41(24): 2074—2077.
24. Li Jianping, Chou Jifan, The property of solutions for the equations of large-scale atmosphere with the non-stationary external forcing, Chinese Science Bulletin, 1996, 41(7): 587—590.
25. Li Jianping, Chou Jifan, Existence of atmosphere attractor, Science in China, Ser. D, 1997, 40(2): 215—224.
26. Li Jianping, Chou Jifan, Operator constraint principle for simplifying atmospheric dynamical equations, Chinese Science Bulletin, 2001, 46: 1053—1056.
27. Li Jianping, Chou Jifan, Inertial manifold of the atmospheric equations, Science in China, Ser. D, 1999, 42(4): 369—377.
28. Li Jianping, Chou Jifan, Computational stability of the forced dissipative nonlinear atmospheric equations, Chinese Science Bulletin, 1999, 44(10): 949—951.
29. Li Jianping, Chou Jifan, Asymptotic behavior of the solutions of atmospheric equations with the topographic effect, Progress in Natural Science, 1999, 9(10): 781—791.
30. Liu Shida, Luo zhexian, Luo Dehai, et al., Progress in the study of nonlinear atmospheric dynamics in China (1995—1998), In 1995—1998 China National Report on Meteorology and Atmospheric Sciences, Beijing: China Meteorological Press, 1999, 155—159.
31. Li Jianping, Zeng Qingcun, Chou Jifan, Computational uncertainty principle in nonlinear ordinary differential equations I, Numerical results, Science in China, Series E, 2000, 43(5): 449—460.
32. Li Jianping, Zeng Qingcun, Chou Jifan, Computational uncertainty principle in nonlinear ordinary differential equations II, Theoretical analysis, Science in China, Series E, 2001, 44(1): 55—74.
33. Zeng Qingcun, Adjustment process and evolution process of atmosphere I, Acta Meteor. Sinica (in Chinese), 1963, 33(2): 163—174.
34. Zeng Qingcun, Adjustment process and evolution process of atmosphere II, Acta Meteor. Sinica (in Chinese), 1963, 33(3): 281—289.
35. Richardson, L. F., Weather Prediction by Numerical Process, London: Cambridge University Press, 1922, reprinted Dover, New York, 1965, 236.
36. Charney, J. G., On the scale of atmospheric motion, Geophys. Publ., 1948, 17(2): 1—17.
37. Charney, J. G., Fjϕrtoft, R., von Neuman, J., Numerical integration of the barotropic vorticity equations, Tellus, 1950, 2: 237—254.
38. Lorenz, E. N., Energy and numerical weather prediction, Tellus, 1960, 12: 364—373.
39. Shuman, F. G., Hovermale, J. B., A six-level primitive equation model, J. Appl. Meteor., 1968, 7: 525—531.
40. Liao Dongxian, Design of Atmospheric Numerical Model (in Chinese), Beijing: China Meteorological Press, 1999, 291.

(Received July 29, 2002; accepted December 31, 2002)